Renormalizability and searching for Z' gauge bosons

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New observables for searching for Z' virtual states in processes $e^+e^- \to \bar{f}f$ are proposed. They are based on the relations following due to renormalizability of an underlying theory between the parameters of the low-energy effective Lagrangian. Two types of the Z' interactions are considered. The observables allow to pick up uniquely the signals of both Z' bosons. The search for the Z' at future linear colliders and the treating of the LEP data are discussed.

1. INTRODUCTION

Among various objectives of the recently finished LEP experiments an important place was devoted to searching for signals of new physics beyond the energy scale of the standard model (SM). Reports on these results are adduced partly in the literature [1]. In the present note we are going to discuss the problem of searching for the heavy Z' gauge boson. This particle is a necessary element of the different models extending the SM. Low limits on its mass following from the analysis of variety of popular models are found to be in the energy intervals 500–2000 GeV [1] (see Table 1 which reproduces Table 9 of Ref. [1]). As it is seen, the values of $m_{Z'}$ (as well as the parameters of interactions with the SM particles) are strongly model dependent. Therefore, it seems reasonable to find some model independent signals of this particle. To elaborate that general principles of field theory must be taken into consideration giving a possibility to relate the parameters of different scattering processes. Then, one is able to introduce variables, convenient for the model independent search for Z' (or other heavy states). These ideas were used in [2] in order to introduce the model-independent sign definite variables for Z' detection in scattering processes with $\sqrt{s} \simeq 500$ GeV.

As it has been pointed out in [2], some parameters of new heavy fields can be related by using the requirement of renormalizability of the underlying model remaining in other respects unspecified. The relations between the parameters of new physics due to the renormalizability were called the renormalization group (RG) relations. In Ref. [2] the RG relations for the low-energy couplings of the SM fields to the heavy neutral gauge boson (Z' boson) have been derived. They predict two possible types of Z' particles, namely, the chiral and the Abelian Z' ones. Each type is described by a few couplings to the SM fields. Therefore, taking into account the RG correlations between the Z' couplings, one is able to introduce observables which uniquely pick up the Z' virtual state [2]. In the present paper we discuss these observables and

Table 1 95% confidence level lower limits on the Z' mass for χ , ψ , η , L–R models [3] and for the Sequential Standard Model (SSM) [4].

Model	χ	ψ	η	L-R	SSM
$m_{Z'}^{ m limit}, { m GeV}/c^2$	630	510	400	950	2260

the possibility of searching for Z' at the present day and future e^+e^- colliders.

2. Z' COUPLINGS TO FERMIONS

As it was pointed out in Ref. [2], to derive RG correlations between the parameters of the Z^\prime interactions with light particles one must specify the model describing physics at low energies. For instance, the minimal SM with the one scalar doublet can be chosen. However, due to the lack of information about scalar fields, models with an extended set of light scalar particles can be also considered. Below we choose the two-Higgs-doublet model (THDM) [5] as the low-energy theory (notice, the minimal SM is the particular case of the THDM).

To derive the RG relations one has to introduce the parametrization of Z' couplings to the SM fields. Since we are going to account of the Z' effects in the low-energy $e^+e^- \to \bar{f}f$ processes in lower order in $m_{Z'}^{-2}$, the linear in Z' interactions with the SM fields are of interest, only. The renormalizability of the underlying theory and the decoupling theorem [6] guarantee the dominance of renormalizable Z' interactions at low energies. The interactions of the non-renormalizable types, generated at high energies due to radiation corrections, are suppressed by the inverse heavy mass. The SM gauge group $SU(2)_L \times U(1)_Y$ is considered as a subgroup of the underlying theory group. So, the mixing interactions of the types $Z'W^+W^-$, Z'ZZ, ... are absent at the tree level. Such conditions are usually used in the literature [7] and lead to the following parametrization of the linear in Z' low-energy vertices:

$$\mathcal{L} = \sum_{i=1}^{2} \left| \left(D_{\mu}^{\text{ew},\phi} - \frac{i\tilde{g}}{2} \tilde{Y}(\phi_{i}) \tilde{B}_{\mu} \right) \phi_{i} \right|^{2} +$$

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$$i\sum_{f=f_L,f_R} \bar{f}\gamma^{\mu} \left(D_{\mu}^{\text{ew},f} - \frac{i\tilde{g}}{2} \tilde{Y}(f) \tilde{B}_{\mu} \right) f, \tag{1}$$

where ϕ_i (i=1,2) are the scalar doublets, \tilde{g} is the charge corresponding to the Z' gauge group, $D_{\mu}^{\mathrm{ew},\phi}$ and $D_{\mu}^{\mathrm{ew},f}$ are the electroweak covariant derivatives, \tilde{B}_{μ} denotes the massive Z' field before the spontaneous breaking of the electroweak symmetry, and the summation over the all SM left-handed fermion doublets, $f_L = \{(f_u)_L, (f_d)_L\}$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood. Diagonal 2×2 matrices $\tilde{Y}(\phi_i), \tilde{Y}(f_L)$ and numbers $\tilde{Y}(f_R)$ are unknown Z' generators characterizing the model beyond the SM.

The one-loop RG relations for the above introduced Z' vertices (1) have been obtained in Ref. [2]. As it was shown, two different types of the Z' generators are compatible with the renormalizability of the underlying theory. The first type, called the chiral Z', describes the Z' boson which couples to the SM doublets, only. The corresponding generators have the zero traces:

$$\tilde{Y}(\phi_i) = -\tilde{Y}_\phi \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\tilde{Y}(f_L) = \tilde{Y}_{L,f_u} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\tilde{Y}(f_R) = 0.$$
(2)

The second type is the Abelian Z' boson:

$$\tilde{Y}(\phi_i) = \tilde{Y}_{\phi} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\tilde{Y}(f_L) = \tilde{Y}_{L,f} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\tilde{Y}(f_R) = \tilde{Y}_{L,f} + 2T_f^3 \tilde{Y}_{\phi},$$
(3)

where T_f^3 is the third component of the fermion weak isospin. The relations (3) ensure, in particular, the invariance of the Yukawa terms with respect to the effective low-energy $\tilde{U}(1)$ subgroup corresponding to the Z' boson. As it follows from the relations, the couplings of the Abelian Z' to the axial-vector fermion currents have the universal absolute value proportional to the Z' coupling to the scalar doublets.

The derived relations (2)–(3) are model-independent ones. They hold in the THDM as well as in the minimal SM. As it is seen from relations (2)–(3), only one parameter for each SM doublet remains arbitrary. The rest parameters are expressed through them. A few number of independent Z' couplings gives the possibility to introduce the observables convenient for detecting uniquely the Z' signals in experiments. In what follows, we consider the searching for the Z' at future linear e^+e^- colliders and treating the obtained at LEP data taking into account the RG relations (2)–(3).

3. Z' SIGNALS AT FUTURE e^+e^- COLLIDERS

The model-independent Z' searches at e^+e^- colliders are intensively discussed in the literature (see, for in-

stance, the reports [8, 9]). The LEP experiments at energies $\sqrt{s} \simeq 200 \text{ GeV}$ were completed recently. The analysis of the obtained data will constrain, in particular, the possible Z' signals. In this regard, the RG relations (2)–(3) can be used to introduce the observables convenient for the detection of the Z' signals in the electron-positron annihilation into the fermion pairs [2, 10]. Before discussing the LEP case, we consider the observables for future linear colliders with the center-of-mass energy $\sqrt{s} \geq 500$ GeV. At energies $\sqrt{s} \geq 500$ GeV the observables can be simplified significantly, since the Z-boson mass can be neglected and all the Z' effects can be described by the four-fermion contact interactions induced by the heavy Z'exchange. The magnitude of these contact interactions is independent of the center-of-mass energy, therefore any value of \sqrt{s} gives predictions close to the $m_Z \to 0$.

Consider the process $e^+e^- \to V^* \to \bar{f}f$ $(f \neq e,t)$ with the neutral vector boson exchange (V=A,Z,Z'). We assume the non-polarized initial- and final-state fermions. Since the t quark is not considered, the fermions can be treated as massless particles, $m_f \sim 0$. In this approximation the left-handed and the right-handed fermions can be substituted by the helicity states, which we mark as λ and ξ for the incoming electron and the outgoing fermion, respectively $(\lambda, \xi = L, R)$.

The differential cross section of the process $e^+e^- \to V^* \to \bar{f}f$ deviates from its SM value by a quantity of order $m_{Z'}^{-2}$:

$$\Delta \frac{d\sigma_f}{d\cos\theta} = \frac{1}{16\pi s} \operatorname{Re} \left[\mathcal{A}_{SM}^* \right] \times \left(\mathcal{A}_{Z'} + \frac{d\mathcal{A}_Z}{d\theta_0} \Big|_{\theta_0 = 0} \theta_0 \right) ,$$

$$\mathcal{A}_{SM} = \mathcal{A}_A + \mathcal{A}_Z(\theta_0 = 0),$$
(4)

where θ denotes the angle between the momentum of the incoming electron and the momentum of the outgoing fermion, A_V is the Born amplitude of the process, and $\theta_0 \sim m_W^2/m_{Z'}^2$ is the Z-Z' mixing angle. The leading contribution comes from the interference between the Z' exchange amplitude, $A_{Z'}$, and the SM amplitude, $A_{\rm SM}$, whereas the Z-Z' mixing terms are suppressed by the additional small factor m_Z^2/s . Notice that the deviation $\Delta d\sigma_f/d\cos\theta$ depends on the center-of-mass energy through the quantity m_Z^2/s , only.

To take into consideration the correlations (2), (3) let us introduce the observable $\sigma_f(z)$ defined as the difference of cross sections integrated in some ranges of the scattering angle θ , which will be specified below [2, 10]:

$$\sigma_f(z) \equiv \int_z^1 \frac{d\sigma_f}{d\cos\theta} d\cos\theta - \int_{-1}^z \frac{d\sigma_f}{d\cos\theta} d\cos\theta$$
$$= \sigma_f^T \left[A_f^{FB} \left(1 - z^2 \right) - \frac{z}{4} \left(3 + z^2 \right) \right], \tag{5}$$

where z stands for the boundary angle, σ_f^T denotes the total cross section and A_f^{FB} is the forward-backward asymmetry of the process. The idea of introducing the z-dependent observable (5) is to choose the value of the

kinematic parameter z in such a way that to pick up the characteristic features of the Z' signals. In the next sections we will consider the deviations from the SM predictions, $\Delta \sigma_f(z)$, induced by the chiral and the Abelian Z' bosons.

3.1. Chiral Z'

In case of the chiral Z' the deviation $\Delta \sigma_f(z)$ can be expressed as

$$\Delta \sigma_f(z) = \frac{\alpha_{\text{em}} \tilde{g}^2}{32 m_{Z'}^2} \left(\mathcal{G}_1^f(z, s) \tilde{Y}_{L,f} \tilde{Y}_{L,e} + \mathcal{G}_2^f(z, s) \tilde{Y}_{L,f} \tilde{Y}_{\phi} + \mathcal{G}_3^f(z, s) \tilde{Y}_{L,e} \tilde{Y}_{\phi} \right), \tag{6}$$

where $\alpha_{\rm em}$ is the fine structure constant, and $\mathcal{G}_i^f(z,s)$ are the factors dependent on the SM parameters, only. The leading contribution to the quantity $\Delta\sigma_f(z)$ comes from the first term in the sum, since the functions $\mathcal{G}_2^f(z,s)$ and $\mathcal{G}_3^f(z,s)$ originate from Z-Z' mixing and contain the additional small factor m_Z^2/s . So, for the center-of-mass energies $\sqrt{s} \geq 500$ GeV the deviation $\Delta\sigma_f(z)$ is computed to within 2% in the form:

$$\Delta \sigma_f(z) \simeq \frac{\alpha_{\text{em}} \tilde{g}^2}{32m_{Z'}^2} \mathcal{G}_1^f(z, s) \tilde{Y}_{L,f} \tilde{Y}_{L,e},$$

$$\mathcal{G}_1^f(z, s) \simeq \frac{4T_f^3 N_f}{3} \left(1 + |Q_f| + \ldots\right)$$

$$\times \left(1 - z - z^2 - \frac{z^3}{3}\right), \tag{7}$$

where N_f is the number of colors, Q_f is the fermion charge in the positron charge units, and dots stand for the terms of order $O(1-4\sin^2\theta_W, m_Z^2/s)$ (θ_W is the Weinberg angle).

As it is seen, the quantity $\Delta \sigma_f(z)$ is proportional to the same polynomial in z for any final fermion state. As a consequence of these factorization, the observable $\Delta \sigma_f(z)$ is completely determined by the deviation of the total cross section. This is the distinguished property of the chiral Z' signal.

Comparing the deviations $\Delta \sigma_f(z)$ for the fermions which belong to the same left-handed isodoublet, $\{f_u, f_d\}$, one finds that the ratio $\Delta \sigma_{f_u}(z)/\Delta \sigma_{f_d}(z)$ is independent of the boundary angle z. It equals to 5/4 for quarks and 1/2 for leptons in lower order in small parameters $1-4\sin^2\theta_W\simeq 0.08$ and m_W^2/s .

 $1-4\sin^2\theta_W\simeq 0.08$ and m_W^2/s . At $z=2^{2/3}-1\simeq 0.5874$ the quantity $\Delta\sigma_f(z)$ becomes zero. In other words, the chiral Z' signal can be kinematically suppressed by choosing this value of the boundary angle. On the other hand, among the deviations $\Delta\sigma_f(z)$ computed at different boundary angles z, the deviation of the total cross section is maximal.

3.2. Abelian Z'

Now, let us consider the observables for the Abelian Z' signals at future linear colliders. In this case the quantity

(5) can be written as follows

$$\Delta \sigma_{f}(z) = \frac{\alpha_{\text{em}} \tilde{g}^{2}}{16m_{Z'}^{2}} \left(\mathcal{F}_{0}^{f}(z, s) a_{Z'}^{2} + \mathcal{F}_{1}^{f}(z, s) v_{Z'}^{f} v_{Z'}^{e} + \mathcal{F}_{2}^{f}(z, s) v_{Z'}^{f} |a_{Z'}| + \mathcal{F}_{3}^{f}(z, s) v_{Z'}^{e} |a_{Z'}| \right).$$
(8)

where $v_{Z'}^f \equiv (\tilde{Y}_{L,f} + \tilde{Y}_{R,f})/2$ and $a_{Z'}^f \equiv (\tilde{Y}_{R,f} - \tilde{Y}_{L,f})/2 = T_f^3 \tilde{Y}_\phi$ are the Z' couplings to the vector and the axial-vector fermion currents. Functions $\mathcal{F}_i^f(z,s)$ are determined by the SM quantities and depend on the fermion type throw the charge Q_f and the number of colors N_f , only. They are independent of the fermion generation. The leading contributions to the lepton factors $\mathcal{F}_2^l(z,s) = \mathcal{F}_3^l(z,s)$ equal to zero. So, by choosing the boundary angle z^* to be the solution to the equation $\mathcal{F}_1^l(z^*,s) = 0$, one can switch off three lepton factors $\mathcal{F}_i^l(z,s)$ (i=1,2,3) simultaneously. Note that the function $z^*(s)$ is the decreasing function of energy. This is shown in Fig. 1 for energies $\sqrt{s} = 200$ –700 GeV.

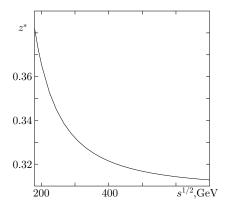


Figure 1. z^* as the function of the center-of-mass energy.

By choosing $z = z^*$, we obtain the sign definite observable

$$\Delta \sigma_{l}(z^{*}) \simeq \frac{\alpha_{\text{em}} \tilde{g}^{2}}{16m_{Z'}^{2}} \mathcal{F}_{0}^{l}(z^{*}, s) a_{Z'}^{2}$$

$$\simeq -0.10 \frac{\alpha_{\text{em}} \tilde{g}^{2} \tilde{Y}_{\phi}^{2}}{16m_{Z'}^{2}} < 0. \tag{9}$$

The quantity $\Delta \sigma_l(z^*)$ is negative and the same for the all types of SM charged leptons. This is the model-independent signal of the Abelian Z' boson.

For the energy values in the range $\sqrt{s} \ge 500$ GeV one also is able to introduce the sign definite observables for the quarks of the same generation:

$$\Delta \sigma_q(z^*) \equiv \Delta \sigma_{q_u} + 0.5 \Delta \sigma_{q_d}$$

$$\simeq 2.45 \Delta \sigma_l(z^*) < 0. \tag{10}$$

Thus, the signal of the Abelian Z' boson can be detected from the quark variables $\Delta\sigma_{q_u}(z^*)$ and $\Delta\sigma_{q_d}(z^*)$ which are dependent. Moreover, they are related to the lepton observable $\Delta\sigma_l(z^*)$. Possible values of the quark variables in the plane $\Delta\sigma_{q_u}(z^*)$ – $\Delta\sigma_{q_d}(z^*)$ are to be at a straight line crossing the axes in the points $\Delta\sigma_{q_u}(z^*)=2.45\Delta\sigma_l(z^*)$ and $\Delta\sigma_{q_d}(z^*)=4.9\Delta\sigma_l(z^*)$, respectively (see Fig. 2). As the chiral Z' boson is concerned, experimental data have to be at some point on the different straight line $\Delta\sigma_{q_u}(z^*)=5\Delta\sigma_{q_d}(z^*)/4$. This fact is very important distinguishable feature of the variables $\Delta\sigma_l(z^*)$, $\Delta\sigma_q(z^*)$, which select the Z' boson signals in the processes $e^+e^-\to \bar f f$. As a result, introduced observables are perspective in the model-independent searching for the Z' signals at future linear colliders.

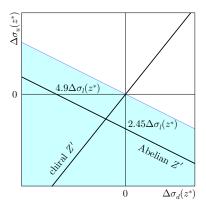


Figure 2. Signals of the chiral and the Abelian Z' in the plane of the observables for quarks of the same generation. The shaded area represents the Abelian Z' signal for all possible values of the axial-vector couplings $a_{Z'}^f$.

4. Z' SIGNALS AT LEP ENERGIES

Now, let us discuss the applicability of the introduced observables to analyse the LEP data. At the LEP center-of-mass energies $\sqrt{s} \sim 200$ GeV the effects of the Z–Z' mixing become more pronounced and the observables (6) and (8) become more complicated. In what follows, we consider the lepton observables $\Delta \sigma_l(z)$, only. Some lepton factors $\mathcal{G}_i^l(z)$ and $\mathcal{F}_i^l(z)$, entering the quantities (6) and (8), are dependent: $\mathcal{G}_2^l(z,s) = \mathcal{G}_3^l(z,s)$ and $\mathcal{F}_2^l(z,s) = \mathcal{F}_3^l(z,s)$. In Figs. 3–4 we compare the factors for the chiral Z' and the Abelian Z' computed at the energies $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV.

As it is seen, at the LEP energies the contributions of the chiral factors $\mathcal{G}_2^l(z)=\mathcal{G}_3^l(z)$ are of order 10–15% of $\mathcal{G}_1^l(z)$. Therefore, they cannot be omitted, as it was resonable done for energies $\sqrt{s}=500$ GeV. These contributions spoil the factorization of the dependence on the

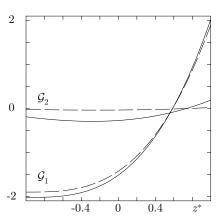


Figure 3. The lepton factors \mathcal{G}_1^l , $\mathcal{G}_2^l = \mathcal{G}_3^l$, entering the chiral Z' observables $\Delta \sigma_l(z)$, computed at the center-of-mass energies $\sqrt{s} = 200$ GeV (the solid curves) and $\sqrt{s} = 500$ GeV (the dashed curves).

boundary angle z for the observable $\Delta \sigma_l(z)$. Therefore, the proportionality of $\Delta \sigma_l(z)$ to the total cross-section deviation $\Delta \sigma_l^T$ does not hold.

In Fig. 4 we show the factors $\mathcal{F}_i^l(z,s)$ computed at energies $\sqrt{s}=200$ and 500 GeV. As it is occurred, the only factor $\mathcal{F}_0^l(z)$ at a_Z^2 , changes significantly. Other functions $\mathcal{F}_1^l(z,s)$, $\mathcal{F}_2^l(z,s)=\mathcal{F}_3^l(z,s)$ contribute less than 2%. Hence, as the Abelian Z' is concerned, one is able to introduce the sign-definite lepton observables $\Delta\sigma_l(z^*)$ at the LEP energies. These observables differ from ones at $\sqrt{s}=500$ GeV by the larger value of the boundary angle $z^*=0.366$ ($z^*=0.317$ at $\sqrt{s}=500$ GeV).

In fact, the quantities $\Delta \sigma_l(z^*)$ depend on one unknown Z' parameter, $\tilde{g}^2 \tilde{Y}_{\phi}^2/m_{Z'}^2$, only:

$$\Delta \sigma_l(z^*) \simeq -0.528 \frac{\alpha_{\rm em} \tilde{g}^2 \tilde{Y}_{\phi}^2}{16m_{Z'}^2}.$$
 (11)

Moreover, this parameter determines the deviation of the ρ -parameter, $\rho \equiv m_W^2/(m_Z^2 \cos^2 \theta_W)$, from unit [2, 10]:

$$\frac{\tilde{g}^2 \tilde{Y}_{\phi}^2}{m_{Z'}^2} \simeq \frac{4\pi \alpha_{\rm em}}{m_W^2 \sin^2 \theta_W} \left(\rho - 1\right). \tag{12}$$

As a result, the Abelian Z' signal is described by the model-independent relations:

$$\Delta \sigma_{\mu}(z^*) \simeq \Delta \sigma_{\tau}(z^*)$$

$$\simeq -\frac{0.528\pi\alpha_{\rm em}^2}{4m_W^2\sin^2\theta_W} (\rho - 1) < 0. \tag{13}$$

Hence, there are three different sign-definite observables to measure only one parameter of the Abelian Z' boson. This one-parametric dependence of the Abelian Z' signal gives a possibility to derive severe model-independent constraints on the Z' couplings. Thus, the observables

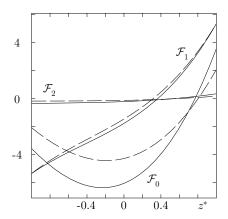


Figure 4. The lepton factors \mathcal{F}_0^l , \mathcal{F}_1^l , $\mathcal{F}_2^l = \mathcal{F}_3^l$, entering the Abelian Z' observables $\Delta \sigma_l(z)$, computed at the center-of-mass energies $\sqrt{s} = 200$ GeV (the solid curves) and $\sqrt{s} = 500$ GeV (the dashed curves).

 $\Delta \sigma_l(z^*)$ were found to be the most sensitive for the Abelian Z' searches when one treats the LEP data.

The present investigation shown that the necessary condition of the renormalizability of the underlying theory, formulated in relations (2) and (3), gives the possibility to determine a number of specific correlations between different scattering processes and to introduce the model-independent observables convenient in searching for signals of the chiral and the Abelian Z' bosons both at future e^+e^- colliders and at energies of LEP.

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